ENVISIONING BETTER MATH LEARNING EXPERIENCES:
The Role of Character Education and the Common Core State Standards for Mathematics (Middle School)
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Introduction/context: We establish the general need for a fresh look at mathematics, identify key issues, and frame character education as an essential tool for addressing these issues. Specifically, we show that while the United States has had more than 60 years of concern about academic standards, math still has an ambiguous role in our culture and a real but perhaps unnecessary anxiety producing effect in schools. On the one hand, strong capacity in math is seen at the individual level as essential preparation for a range of careers. Also, our national mathematics capacity is seen as a tool for ensuring our global economic competitiveness. Balancing these strongly pro-math values is the negativity produced at the individual level by the “gate-keeper” status of various math courses as well as general malaise about math in the broader society. As mathematician John Allen Paulos famously noted, it’s OK at a cocktail party to say that you’re not a math person, whereas you would never say that you just “don’t get” language. In short, we value math, but we don’t really like it.

Against this background, the United States is embarking on our third effort to create meaningful change through national standards. Despite ostensibly good intentions, it is a fair question to ask whether this effort will succeed where previous ventures have not. In this paper, we take the position that without due attention to factors beyond curriculum change, the mathematical expertise targeted in the Common Core will be thinly realized at best. More specifically, we argue that various aspects of character are explicitly identified (perseverance, cooperation, precision, creativity), while other aspects are implicitly assumed (empathy, confidence, fairness, courage), within the Common Core vision, and use this paper to articulate their role (and advocate for their inclusion) in math reform discussions.

Position taken: For teachers, students, and community members to embrace and realize the vision of the Common Core Standards, a fundamental shift will be required in how we envision teaching and learning mathematics. Merely adjusting the curriculum sequence and holding workshops to orient teachers to the new standards will amount to what Tyack and Cuban have called “tinkering toward Utopia.” Efforts to double down on business-as-usual, even with increased rewards and consequences, will not lead to significant change. Instead, we need to build confidence, intrinsic interest, growth
focused orientations around mathematics, and around learning in general – in children, teachers and parents. In this paper we identify specific research supported interventions for building these character strengths and increasing academic performance in math. We articulate ways in which constructs developed within the character education field hold the key to a more vibrant mathematical learning environment.

**Specific actions recommended:** We advocate the creation of a Dewey-inspired character-building model of “Communities of Inquiry” toward mathematical tasks that moves beyond training students to replicate procedural knowledge. Done well, this establishes a culture of engagement and intrigue with diverse strands of mathematics. Countering the obvious objection, we show how participation in a community of inquiry leads to higher real achievement, both for individuals and groups. Within a community of inquiry, the scope of work is defined by the practice and content standards, while norms of group membership are defined by concerns for personal, civic, performance, and intellectual character.

To make this vision tangible, we present four extended vignettes (primary, intermediate, middle, and high school) exemplifying how a richer learning space emerges through the intersection of character education, strong pedagogy, and the Common Core Standards. Complementing the classroom environment, we look at ways in which the community beyond the classroom can be a resource for students needing additional support or enrichment, and how parents can be engaged in these changes. We include 8 detailed lesson plans aligned to each of the CCSS-M practice standards and the encompassed virtue that also include opportunities for reflection by students, teachers, and families. We also articulate the relevant administrative, assessment, and pedagogic issues raised by our character education infused model, and offer a research and development agenda to guide a path forward.

**Searching for a Better Math Learning Experiences: The Role of Character Education**

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The mathematics education community across the United States is currently embarking on its third reinvention in just over two decades. Sparked by a flurry of reform efforts in the aftermath of the landmark *Nation at Risk* report (National Commission on Excellence in Education, 1983) which in its call to action famously declared “If an unfriendly foreign power had attempted to impose on America the mediocre educational performance that exists today, we might well have viewed it as an act of war. As it stands, we have allowed this to happen to ourselves...We have, in effect, been committing an act of unthinking, unilateral educational disarmament.” In the years following this, math education reform was initiated with the *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, 1989). These voluntary standards were developed by NCTM to guide classroom practice, teacher professional development, and student evaluation. Consisting of standards
grouped in grade bands for K–4, 5–8, and 9–12, these standards promoted a rigorous and challenging view of mathematics to which all students should strive. While there was great discussion and enthusiasm among supporters, widespread change didn’t materialize. Undaunted, NCTM followed up in 2000 with the *Principles and Standards for School Mathematics*, which offered a revised set of standards grouped in a new set of grade bands (K–2, 3–5, 6–8, and 9–12). These new standards promoted another view of a rigorous and challenging view of mathematics to which – again – all students should strive. Along with these documents put forth by the preeminent mathematics teacher organization in the United States, there were other reform-oriented documents published, such as *Before it’s Too Late: A Report to the Nation* (National Commission on Mathematics and Science Teaching for the 21st Century, 2000), and *Adding it Up*, a National Academies of Science (Kilpatrick et al, 2001) report on math education in grades K–8. Like the NCTM documents, these reports tried valiantly to show the urgency of math education reform.

Still, lasting change continued to prove elusive, leading to the development of the *Common Core State Standards for Mathematics* (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). This latest iteration of reform offers schools a new set of standards, again broken into grade bands with the intention of guiding classroom practice, teacher professional development, and student evaluation, though this time with a claim of refining the effort toward fewer, more coherently aligned standards than were found in previous standards efforts. The refrain in these opening paragraphs is intentional, as each iteration of standards changes a few verses, but is strategically very similar to those before it. The operating assumption is that by laying out a path of recommended action, the educational community will flock to it, leading to improved learning for all. Key differences in the latest round are (1) its state-wide adoption by many state departments of education or the equivalent body, and (2) the close alignment of the Common Core with two authorized student testing programs. As of this writing, 45 of 50 states, the District of Columbia, four territories, and the Department of Defense schools have formally adopted the Common Core curriculum. Thus, even though nominally chosen independently by each state, the nearly universal governmental backing and common assessment framework has the effect of being official national standards rather than simply being a set of recommendations from a professional body.

An open question at this point is whether the addition of the state (and de facto national) imprimatur placed on the Common Core will lead to better learning experiences than were realized under the previous standards put forth by the NCTM. We are skeptical, believing that without a cultural change to accompany the new standards, change will be at the surface level at best. To realize more substantive reform
of students’ mathematics learning opportunities, an approach that draws on and builds a range of character attributes is essential. In the pages ahead, we will offer an overview of the underlying (arguably dysfunctional) view of mathematics prevalent in schools today, and show how an alternative approach infused with character and moral imagination offers a path forward.

**What’s the Problem?**

Given the flurry of reform activity sketched in the previous section, it’s worth looking at the underlying problem to get a better handle on why math education continues to be a hot spot in our continuing quest for educational reform. There are two key issues to address in this section which provide signposts to guide improvements: (1) students’ relatively low performance in mathematics when compared to their peers internationally, and (2) the extent to which students distance themselves from mathematics learning. We’ll consider each in turn.

Starting with the landmark report *A Nation at Risk* noted earlier and continuing to today, there is a great concern about the impact low achievement has on our national economic competitiveness. While this concern is most often manifested in images of not having enough scientists and engineers to support research and innovation, low achievement also plays out in line workers who are no longer sufficiently educated for functioning in a manufacturing environment that relies on advanced monitoring and controls. Ripley (2013) cites the example of an Oklahoma factory that makes the apple pies for McDonald’s. Despite the recession, the owner couldn’t staff his factories with high school and community college graduates, and opened a new factory in Guangzhou, China. Similarly, Ripley cites the challenges that Manpower (the temporary services agency) has in staffing sales jobs. No longer simply the domain of positive personal relationships, salespeople need to understand and communicate sufficiently well the complexities and customization options of the products they are selling. Low fluency with relationships and reasoning – bedrock components of mathematics – affects all levels of the economy.

Just how bad are we? Looking at the data, students in the United States don’t perform poorly, but as beneficiaries of the world’s largest economy (and thus having the resources to support a strong educational system), we could be doing better. For the most recent Trends in International Mathematics and Science Study (TIMSS) – generally seen as a benchmark for international comparisons in mathematics – fourth graders in the United States placed 11th, clearly behind 7 nations and without a statistically significant difference when compared to three others nominally ahead of the United States. The top five are Asian countries with advanced economies (Singapore, Republic of Korea, Hong Kong, Chinese Taipei, and Japan). Others ranking clearly
higher than the United States are Northern Ireland and Flemish Belgium. Our statistical peers include Finland, England, the Russian Federation, the Netherlands, and Denmark.

Moving on to eighth grade, a slightly different mix of nations are represented, but the United States maintains its solid but hardly stellar performance. The five Asian countries noted in the fourth grade results remain ahead of us, and the Russian Federation has advanced past us by a statistically significant margin. Our statistical peers in eighth grade include Israel, Finland, England, Hungary, Australia, Slovenia, and Lithuania.

Taken together, these comparative results suggest two points worth considering: First, much of our concern with economic competitiveness centers around Asian markets, where the TIMSS results shows them having a clear achievement advantage. Second, the eighth grade results show that there are countries with considerably fewer resources such as Slovenia and Lithuania which are performing at a level comparable to that of the United States. An additional concern specific to the United States is that our score in eighth grade is regressing toward the test’s 500 scale average. Whereas in fourth grade we scored 541, by eighth grade the score had slipped to a more middling 509.

Why do students in the United States not perform as well as others? There are likely a number of reasons. A primary one, we argue, is that students feel greatly distanced from math as they encounter it in school. It is hard to sustain interest, focus, and investment in an endeavor if it leaves you indifferent at best. Once this dynamic is in play, policymakers from state and district administrators down to the individual classroom teacher and parent begin a carrot and stick game to encourage mathematical performance. For students, it’s all too often just something to get through to achieve a greater goal such as graduation or to avoid disciplinary consequences. Nowhere is there authentic engagement. Instead, in the language of self-determination theory (Deci & Ryan, 2002), it is an introjection, externally imposed and dealt with until it goes away. Continuing in that vein, we need instead to build toward students taking on the identity of self determined learners who exhibit autonomy, competence, and reliability. An “indifferent at best” approach toward math will confound any efforts at improvement since students will remain passive participants.

In addition to the achievement angle, it’s also worth considering the ways in which students distance themselves from math as it is presented in a typical classroom. As Paulos (2001) notes, saying that “I’m just not a math person” is a common and culturally acceptable identity, which we believe both reflects and feeds back into seeing math as an alien, remote field. This contrasts with math as it is used for real and
personally valued purposes, reflected everywhere from street vendors who can make change with great fluency, to supermarket shoppers who can handle unit pricing and discounts, to ten year old kids playing video games which require rich data analysis for successful game play. If we are to reinvigorate math education – whether it be for reasons of national economic progress or just because it can be a fascinating field – we need to address this distancing phenomenon and return math to its proper place as a very human endeavor on par with the arts, literature, and the humanities.

As measures of students’ feelings toward math class, we can start with research showing that fifth graders felt that they needed a teacher to guide them through the mysteries of mathematics, whereas they felt that social studies could be learned independently (Stodolsky et al., 1991). Countering this, Kamii and Housman (1999) and Fosnot and Jacobs (2010) have demonstrated that kids can in fact build mathematical knowledge, albeit in a very different learning environment than the typical math classroom. Schoenfeld’s research (1989) raises a further concern, showing that many students see math ability as innate. This line of research has great overlap with extensive research by Carol Dweck (2000) on fixed vs. changeable mindsets, finding in general that a fixed mindset can limit students’ learning horizons. Within a fixed mindset, there is often a perception along the lines of “If I’m just not good at it and can’t do anything about it, why bother trying?” Neither apathy nor feeling incompetent is helpful in building a positive regard for math as both an interesting and useful endeavor. Instead, too many students distance themselves from mathematical sense-making.

In practice, this distancing has to be a significant factor in middling test results and a cause of many students dropping math when they can. Looking at performance, De Corte et al (2010) cite examples from the research literature showing how students faced with a math “story problem” develop absurd pro forma answers detached from any real context. For example, they cite a French example which explicitly tested for this lack of attention to the task at hand: “There are 26 sheep and 10 goats on a boat. How old is the captain?” While there is no way to know this from the information given, students dutifully answered, most often with “36” as the answer. Through answers like these, students give life to the parody of problem solving that has been circulating in the math education field for decades:

_A Student’s Guide to Problem Solving_

Rule 1: If at all possible, avoid reading the problem. Reading the problem only consumes time and causes confusion.

Rule 2: Extract the numbers from the problem in the order in which they appear.
Be on the watch for numbers written in words.

Rule 3: If rule 2 yields three or more numbers, the best bet for getting the right answer is adding them together.

Rule 4: If there are only two numbers which are approximately the same size, then subtraction should give the best results.

Rule 5: If there are only two numbers in the problem and one is much smaller than the other, then divide if it goes evenly — otherwise, multiply.

Rule 6: If the problem seems like it calls for a formula, pick a formula that has enough letters to use all the numbers given in the problem.

Rule 7: If rules 1-6 don’t seem to work, make one last desperate attempt. Take the set of numbers found by rule 2 and perform about two pages of random operations using these numbers. You should circle about five or six answers on each page, just in case one of them happens to be the answer. You might get partial credit for trying hard.

Rule 8: Never, never spend too much time solving problems. This set of rules will get you through even the longest assignments in no more than ten minutes with very little thinking.

Even when students do make rational calculations, sense-making isn’t often a part of the problem solving process. De Corte et al. (2010) also cite an example of a simple division problem in which 1,128 soldiers need to be transported on buses, each of which carries 36 people. Only 70% of the middle school students could perform this division, which in itself is a cause for concern to which we will return later. Of those who did generate an answer, the most common response reflected the standard elementary school “long division” response (31 r 12). Only 23% of the students correctly noted that a 32nd bus would be required. Whether it’s an issue of goats on a boat or soldiers left behind, math has been reduced to a formulaic “crunch the numbers and spit back what you get” process. Mathematician-turned-educator Paul Lockhart (2009: 27) cites similar cognitive paralysis using a geometry example before rueing “This is why it is so heartbreaking to see what is being done to mathematics in school. This rich and fascinating adventure of the imagination has been reduced to a sterile set of facts to be memorized and procedures to be followed.” If we are to have improved opportunities to learn mathematics, we need a more well thought out strategy than simply imposing higher standards backed by more rigorous accountability measures. We believe, and advocate in the pages that follow, for an approach to math education that builds on a framework of character attributes that explicitly encourage and reward full personal
engagement with interesting situations, for which mathematical understanding holds the key. Only then can we have a reasonable hope of moving past our current malaise of mathematical apathy and underachievement.

*Dimensions of Character Education*

Before showing examples of how a sustained focus on character development can lead to improved practice in math education, we need to delineate the playing field a bit. To do that, we adopt a framework for character development advanced by David Shields (2011). In his framework, Shields identifies four components of individual character that are worth consideration, as well as an overarching concept of community character that helps in understanding the broader context within which students grow up. As he notes (2011: 49):

> These multiple dimensions of character share a focus on *personal dispositions* and *patterns of interaction*. They focus on constructing meaning and how a person acts in various aspects of their life and learning. The goal of education is not acquiring knowledge alone, but developing the dispositions to seek and use knowledge in effective and ethical ways. [%emphasis original%]

We’ll be considering Shields’ dimensions as they apply specifically to mathematics later, but to start the discussion, we offer in summary form these dimensions of character:

**Moral character:** Phrased by Shields as “a disposition to seek goodness,” moral character involves a disposition to act in ways that are kind, helpful, and respectful of other peoples’ needs and interests. While some try to reduce moral character to a set of specific virtues, a more general and more useful framing is offered by Dewey in his framing of moral values as part of a pragmatic social intelligence (Fesmire, 2003). Here, rather than running each situation through a fixed decision tree, a person faced with moral choices has the situational awareness to understand the relevant issues and make a choice that is appropriate for the time and place. A different set of circumstances may call for a different response. While it may seem odd to consider the moral dimensions of mathematics education, we will see over the next few pages that it might just hold the key to bringing new vitality to the field.

**Civic character:** Responding to John Donne’s dictum that “no man is an island,” strong civic character – phrased succinctly by Shields as “passion for the public good” – equips us with the disposition to be actively engaged in the communities we find ourselves a part of. This might be on a small scale such as a family or a working team, or in progressively larger groups such as the classroom, school, community, and beyond. In each, there is an impetus to be a contributing member of that society both in terms of what we bring to the table and in the ways in which our actions and attitudes serve to
build up the vitality and functional ability of that community. This effort at improving the educational civitas may look out of place in an effort to improve students’ academic achievement (which in the United States tends to be very individually focused), but we will see in the project examples offered later that there is no inherent trade-off that needs to be made between pursuit of individual excellence and being a contributing member of a learning community.

*Performance character:* With performance character, we get into more common concerns expressed in the educational reform literature. Dominant issues here relate to how students persist in working through difficult problems, how willing they are to re-think and re-work problems, and whether students strive for higher-level mathematics or simply “work to script” in meeting minimum graduation requirements. Given the malaise noted earlier in regard to how poorly many students in the United States perceive math, it’s not surprising that there is considerable room for improvement in many students’ performance character. Without finding anything inherently interesting in math, and without a vision for how math as it is learned in school has any practical value, it’s a subject that many students pursue passively and flee as soon as they are able. This attitude is captured in the T-shirt slogan “Well, another day has passed and I didn’t use algebra once.” We’re confident based on research in the math education field and from more general research on learning dispositions that changing the math learning environment and the nature of the mathematical tasks students work on will effect significant change in performance character.

*Intellectual character:* The fourth dimension of personal character Shields cites concerns intellectual attributes such as how we engage in argumentation, handle evidence, and deal with contrasting points of view. Key issues here might be captured under the mantra “resolve within reason” with an emphasis on holding to and defending positions using the intellectual tools at our disposal as far as warranted, but not in such a way that we become deaf to reason. If there is an alternative or complementary argument put forth, we have an intellectual obligation to consider it and debate it on its merits, not simply dismiss it because it came from somewhere else. The inverse of this position, of course, is that intellectual character also requires that we not simply accede to another argument put forth. If we are truly truth-seeking, we will weigh each position fairly on its merits. Doing this well requires a level of mathematical understanding and engagement well past formulaic memorization.

Working in tandem with these dimensions of individual character, Shields describes an overarching consideration for building a culture of character. Here, he is concerned with how the school community in which the students work serves to promote these dimensions of character. In the highly individualistic, test score-based culture of most current mathematics education in this country, it should be clear that considerations of
character are secondary to the driving concern for “improvement” as measured through increased test scores. Thus, character education is all too often shut out or given short shrift in the limited time available in the day. We agree with Shields and many other advocates of character education that we need to be giving more attention to students’ character development as well as their academic achievement. But we go further: In the balance of this paper, we will make the argument that the dualistic “character education vs improved academics” battle for time and attention in the curriculum is misplaced. Rather, the emphasis needs to be on ways in which concern for character is not a competing demand for time, but rather is the medium through which improved mathematics achievement can be realized. Competition between essential educational outcomes is counterproductive. Instead, we propose a path of synergy, leading to a character-rich learning experience which fosters achievement through moral, civic, performance, and intellectual character development.

Our Proposal in Brief

The terrain covered in the preceding pages described a math learning culture which has been the subject of multiple iterations of national-scale attention and reform efforts dating back at least to the early 80’s and in some respects even further. While each effort makes largely similar claims and proposes a similar path toward improvement, progress is scant. Student performance remains middling at best compared to students in other countries with advanced economies (and thus by implication, having the resources to fund educational initiatives). As part of this middling performance, it needs to be recognized that students (and adults by and large) neither like nor value math as it is currently experienced in schools. We need a more significant change in focus and attitude toward mathematics.

As a path toward improved learning opportunities, we will make the case that the current (and historic) approach to math education is miseducative in Dewey’s sense of the word, in that it serves to limit or arrest future development. In its place, we will offer an alternative framing founded on the underlying concept of a community of mathematical inquiry which invites a new perspective on learning and using math. Within this community of inquiry, each of the four dimensions of character described earlier is a pillar in a revitalized approach to mathematics education.

To see such a program in action, let’s peek into very real midwestern third grade classroom to see a few activities:

Students are hard at work on the Four Fours problem, looking for ways to craft equations that use four instances of the number four and any math operations needed. The goal is to find at least one equation for each number from 1 - 100. For example, \(4 + 4 + 4 + 4 = 16\), while \(4 \times 4 + 4 \times 4 = 32\). Add parentheses and you can get \((4 \times 4 + 4)\)
x 4 = 80. Over the couple of weeks in which this problem remains active and prominently displayed on a bulletin board, students learn about parentheses, order of operations, square roots, factorial numbers (4!) and the like. In the daily class meeting, new equations are shared, with appropriate recognition of the mathematicians who crafted them.

Following on this effort, students try to repeat their success with a new challenge: Consecutive Sums. By adding consecutive numbers (e.g. 2+3+4 = 9), can all of the numbers from 1 - 100 be reached? In fact, it can’t be done, but there is a mathematically fascinating pattern in the numbers that can’t be written as the sum of consecutive numbers. How can we describe those numbers? What is it that makes them special and powerful? Students are challenged to work together to describe the patterns as completely as possible, and to predict the next numbers that can’t be written as consecutive sums.

Elsewhere in the classroom, students have a weekly task to record and graph the photoperiod, or more simply, the amount of time between sunrise and sunset. Through this activity, students practice math in different number systems (a day of nominally 24 hours but that is broken into 12 hours in the morning and 12 after noon, as well as hourly cycles of 60 minutes) and in using mathematical tools to record a lived experience of shorter winter days followed by longer spring days.

Over in the corner, students are playing Equations, a game with specialized dice. A goal setter has set a goal of 28, and students take turns placing number and operation cubes on the game mat. A required cube makes that number or operation mandatory for all players to use in their equation, whereas a forbidden cube does the opposite. Players also have the option of permitting a cube on their move, which allows the cube to be used or not as each player desires. While everyone is working toward that goal, the added wrinkle in Equations is that a given player never wants to be the one who allows the goal to be reached by the next player, or to make it impossible to ever reach the goal. What ensures is a dance among players of successively more complex equations.

Finally, it’s not uncommon for class meetings to include short whole-group discussions such as one about which member of a set of numbers (such as 9, 16, 25, and 43) doesn’t belong in the set. While one student suggests that 9 doesn’t belong since it is the only single digit number, another suggests that 16 should be out since it is the only even number. A third perspective casts doubt on 43, since it is the only non-square number. Through developing, articulating, and debating possible answers, students develop an appreciation for others’ perspectives and the richness of mathematical concepts.

What each of these projects has in common is that the students are enthusiastically engaged in higher-level mathematics in a fun and collaborative manner. While “mistakes” will be made, of course, when this happens students are simply invited to
revise their thinking with the help of peers who show other, more effective approaches. Incidentally, the students in this class earned outstanding standardized test scores relative to their peers in previous years who had participated in a more traditional math program.

**Building Character Through Moral Imagination**

As paradoxical as it may seem, the key to revitalized math education may well be through a focused effort to promote the growth of students’ moral imagination. This line of reasoning emerges from a passage in one of John Dewey’s less known works, *Moral Principles in Education* (1909/2009: 33-34). In it, he relates a classroom situation:

Imagine forty children all engaged in reading the same books and in preparing and reciting the same lessons day after day. Suppose this process constitutes by far the larger part of their work, and that they are continually judged from the standpoint of what they are able to take in a study hour and reproduce in a recitation hour. There is next to no opportunity for any social division of labor. There is no opportunity for each child to work out something specifically his own, which he may contribute to the common stock, while he in turn participates in the productions of others. All are set to do exactly the same work and turn out the same products. The social spirit is not cultivated, in fact, in so far as the purely individualistic method gets in its work, it atrophies for lack of use. One reason why reading aloud in school is poor is that the real motive for the use of language – the desire to communicate and to learn – is not utilized. The child knows perfectly well that the teacher and all his fellow pupils have exactly the same facts and ideas before them that he has; and he is not giving them anything at all. And it may be questioned whether the moral lack is not as great as the intellectual. The child is born with a natural desire to give out, to serve. When this tendency is not used, when conditions are such that other motives are substituted, the accumulation of an influence working against the social spirit is much larger than we have any idea of, especially when the burden of work, week after week, and year after year, falls upon this side.

If we simply replace the language example with a typical math class, Dewey is describing quite accurately the ennui that afflicts most math classes in the United States. Instead of a “read and recite” patina, the math class is all too often a churn toward everyone working individually to solve an array of pre-set problems. The “best” students are the ones who can churn the fastest, and over time they are rewarded with the opportunity to churn a greater quantity of even more advanced problems. Rarely is there ownership or a social dimension to school mathematics – one that invites participation, that rewards creativity and innovation in problem solving, and that
enables meaningful interaction among the students and between the teacher and the students.

If we accept Dewey’s claim (1909/2009: 15) that moral ideas are “ideas of any sort whatsoever that take effect in conduct and improve it” and that “immoral ideas are ideas of whatever sort (whether arithmetical or geographical or physiological) which show themselves in making behavior worse than it would otherwise be,” we can sustain the claim that mind-numbing math is fundamentally an immoral, miseducative endeavor in that it turns students off to further engagement and learning. The net result of this negative impact is that by depriving students of full engagement with mathematics, a pallid substitute serves to limit the range of ways by which students can achieve personal fulfillment and best serve the communities they are part of. Conversely, a learning community imbued with the spirit of rich mathematical learning has the potential to inspire students toward a healthier – and arguably more accurate – view of math. This, by way of contrast with math education as it is usually pursued in the United States and elsewhere, serves to advance personal and communal goals, and thus can be deemed moral in Dewey’s sense of the term.

**Fostering Moral Imagination to Build Mathematical Character**

Combining the two sections preceding this, we advance a 2-part argument: (1) that we can build mathematical engagement and understanding on the four pillars of character, and (2) moral imagination – which Mark Johnson defines as “an ability to imaginatively discern various possibilities for acting in a given situation and to envision the potential help and harm that are likely to result from a given action” – is the overlaying fabric which provides unity in the experiences. For example, imagine being a student working on the four fours problem described earlier. While nominally just a computation exercise, for a discerning observer, there is a great deal of character work at stake here as well. To be successful, the student needs to imagine possibilities: What do I know about numbers that will put me in the ball park of my target? Through mental rehearsal using mathematical building blocks (e.g. $4 \times 4$ gets me a 16; the square root of 4 gets me a 2; $4!$ gets me a 24...), different combinations are considered, elaborated upon, and used or rejected as appropriate. In a problem such as this, answers aren’t just given. Rather, they are constructed and evaluated, and may need to be defended to peers as being valid answers. Performance and intellectual character are tested and nurtured as you wrestle with possibilities, as is the very moral question of who you are and what capabilities you are working to develop. Turned around, if another student offers a possible solution, you will need to call on your intellectual character to assess fairly the solution being offered, as well as your civic character as you respond within social norms of friendship and shared inquiry. Of course, moral character rises here as well, in the call to offer kind and constructive feedback. In all, a project such as the Four Fours problem offers more
than just a computation exercise to be graded as correct or incorrect.

Beyond Shields’ four dimensions of character being used here, we see a critically important element of moral imagination at work as well. Taking Dewey’s expansive framing of the moral cited earlier ("ideas of any sort whatsoever that take effect in conduct and improve it"), open ended problems embedded in a creative, engaging, and character-building climate are inherently moral endeavors. As students imagine mathematical and interpersonal possibilities and rehearse what the possibilities are, they delve deeply into the world of the moral imagination. Over time they build the capacity Johnson described as “an ability to imaginatively discern various possibilities for acting in a given situation.” In sum, by restoring the moral imagination which is stripped out of perfunctory math exercises and by giving students many opportunities to relate to themselves and each other in meaningful ways, we can build the mathematical character students need to thrive in the modern world. By extension, since all learning is cumulative in its effects on larger dispositions, character development in the math context is likely to have virtuous impacts on kids’ overall development.

To make this vision of character-rich math learning more tangible, we offer in the next section a series of 8 projects, with 2 keyed to each of the grades 3, 6, 9, and 12. For each, we provide an overview of the activity, its curriculum relevance, and ways in which participating students can use the experience to develop dimensions of character.

References


Sample Projects

In the pages that follow, we offer 8 activities that have the potential to be rich both in mathematical content and in one or more strands of character development as discussed in the paper. We offer two sample activities each for grades 3, 6, 9, and 12. The activities themselves are modifications of projects one might do in a traditional math class (and in fact are drawn from math books). The key differences are in the modifications toward student ownership and creation, and in building a collaborative community of inquiry. As discussed in the paper, the climate of the classroom has to shift toward a character-building environment if students are to be more fully engaged in mathematics as a worthwhile and valued human endeavor. When modified projects such as these become a normal part of the fabric of the school, students will have a much greater chance of successful and satisfactory engagement with math.

Sample Project: Grade 3

Four Fours

Student goal: Students work individually and/or in small teams to generate an equation for each number 1-100 using four instances of the number four and any math operations. Thus, for example, $4 \times 4 \times 4 + 4 = 68$.

Implementation overview: This project lends itself to being introduced either as a whole-class focus for a math activity, or as a challenge that could be the focus for a couple of minutes of time in a class meeting. However it is introduced, the key to it becoming part of a mathematical culture within the classroom is to make it a visible and sustained effort, not a one-off activity. One way to do this is to set up a large display with each of the numbers 1-100 and sufficient space to write a few equations for each. (Space for multiple equations lets students see that there is more than one viable solution for each of the goal numbers.) As students generate equations they can ask peers to verify that the equation leads to the intended goal. This student checking is critical, as it fosters mathematical dialog and peer teaching when the solver uses a technique that is new to the checker, or when mathematical concepts such as order of operations need clarification. A common teaching technique such as "ask three before
“me” can help to keep the discussion focused in peer interaction rather than asking the teacher to check for correctness.

This activity works well for a couple of focused whole-class math periods, after which it can provide a focus for independent work as students seek to complete the table from 1-100. New solutions can be shared in class meetings, offering a chance for recognition of those who persist in finding the more challenging solutions and opportunities for legitimate peripheral participation (Lave and Wenger, 1991) among other class members.

**Character components:** Intellectual character is developed as students learn new mathematical concepts such as square roots, factorials, and order of operations. Performance character is enhanced through an invitation to persistence in working on the harder equations. Civic character is developed as students work to establish and maintain norms of common inquiry and constructive response when they seek clarification or don’t agree, and in their service to others as equation checkers. Moral character is developed as students build their own mathematical identity and by maintaining positive interpersonal relations with their peers and recognizing the talents and achievements of others.

**Extension:** A similar investigation seeking equations from 1-100 using consecutive sums. For example, 14+15+16=45. Not all numbers from 1-100 can be derived this way, but the pattern in which ones can’t sparks interesting mathematical discussion.

**Sample Project: Grade 3**

**Target Numbers**

**Student goal:** Students work to create four numbers using each of the digits 0-9 exactly once. The goal is for the four numbers to have the least total difference from a set of target numbers.

**Implementation overview:** The teacher can post four arbitrary target numbers on the board, along with the digits 0-9 on sticky notes that can be repositioned as students try different options. For example, this solution is fairly close, but could be improved:

<table>
<thead>
<tr>
<th>Target Numbers</th>
<th>Student Answer</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>465</td>
<td>470</td>
<td>5</td>
</tr>
<tr>
<td>389</td>
<td>389</td>
<td>0</td>
</tr>
<tr>
<td>52</td>
<td>52</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>16</td>
<td>15</td>
</tr>
<tr>
<td><strong>Total difference =</strong></td>
<td><strong>20</strong></td>
<td></td>
</tr>
</tbody>
</table>
As with the Four Fours activity, Target Numbers works well for a focused whole-class math period, after which it can be sustained as an independent work option. New solutions can be shared in class meetings, offering a chance for recognition of those who persist and opportunities for legitimate peripheral participation among other class members.

**Character components:** Intellectual character is developed as students reinforce place value and discuss optimization strategies. Performance character is enhanced through an invitation to persistence in working on continuous improvement. Civic character is developed as students work to establish and maintain classroom norms of common inquiry and constructive response to peers. Moral character is developed as students build their own mathematical identity and by maintaining positive interpersonal relations with their peers and recognizing the talents and achievements of others.

**Extension:** Instead of working to minimize the differences, what option maximizes total difference? Can students show that a solution is in fact the best one possible with the given target numbers?

**Sample Project: Grade 6**

**Delivery Days**

**Student goal:** Students work individually and/or in small teams to investigate patterns in three independent delivery routes. Which days have all three deliveries? Two out of three? What patterns can be described?

**Implementation overview:** As students transition beyond arithmetic and toward algebraic thinking, they need to think fluently with multiples and factors. These activities do so in a format that allows students to consider different options and work toward proving mathematically that theirs is the best possible one.

To start the investigation, introduce three delivery drivers: One who arrives every three days, another every four days, and a third who arrives every fifth day. (Students can customize who these drivers are to add a level of personalization.) Students can then be challenged to describe and show using tables and/or graphs when all three drivers will be appearing, as well as when each of the possible pairings will be arriving. What patterns emerge? How can they be described? Informal peer review can be established by pairing students up to see if they have the same answers. Where discrepancies arise, this is fodder for discussion. Returning to the whole-class setting, students can be asked to critique the different ways in which the solutions were presented. Among the norms might be clarity in the information display, aesthetics, and the like.
**Character components:** Intellectual character is developed as students reinforce their understanding of multiples and learn new mathematical concepts such as permutations and consider the pros and cons of different strategies for information display. Performance character is enhanced through working toward a comprehensive solution that accounts for all logical possibilities. Civic character is developed as students work to establish and maintain classroom norms of common inquiry and constructive response when they seek clarification from peers. Moral character is developed as students build their own mathematical identity and by maintaining positive interpersonal relations with their peers and recognizing the talents and achievements of others.

**Extension:** What is the least number into which each of the numbers 1-9 will divide equally? Can students prove mathematically that in fact this is the least number?

**Sample Project: Grade 6**

**Running With The Dogs**

**Student goal:** Students work with a finite amount of fencing to optimize a dog run for the city park.

**Implementation overview:** Students can be presented with a challenge of helping to design a dog run at a local park. Given 100 feet of fence, how would they recommend the dog run be set up? 1’ x 49’ would be one option, giving 49 square feet of space. 2’ x 48’ would almost double this to 96 square feet. Is there a solution that maximizes the area? Is this maximum area the best solution in terms of the dogs’ use of the run? What are the trade-offs that have to be made?

Students can make individual or small group recommendations. As part of their presentation, they can be challenged to decide on the best way to present their data using tools such as graphs and tables. As with the Delivery Days project, students can provide peer review of the different solution options, offering suggestions on key parameters such as clarity of how the information is displayed and the recommendation are made, as well as aesthetic considerations.

As part of the discussion, the students may offer solutions that are based on optimized use, rather than simply maximizing the total area. For example, would a longer, narrow space allow dogs to run better than a more squared up shape? This can lead to discussions where optimal use may not be the mathematically optimal solution.

**Character components:** Intellectual character is developed as students apply and extend mathematical concepts in search of optimization, and as they balance competing
goals (optimal use vs. maximum area). Performance character is enhanced through an invitation to persistence in considering the full range of possible solutions, and as they are challenged to defend theirs as the best solution. Civic character is developed as students apply their mathematics skills to address a community issue, as well as when students work to establish and maintain classroom norms of common inquiry and constructive response when they seek clarification or don’t agree. Moral character is developed as students build their own mathematical identity and by maintaining positive interpersonal relations with their peers and recognizing the talents and achievements of others.

**Extension:** Challenge students to identify other situations where achieving the mathematically maximum result may not be the best overall solution.

**Sample Project: Grade 9**

**Geoboard Detectives**

**Student goal:** To investigate geometric construction challenges with geoboards, with a goal of identifying which challenges are mathematically not possible.

**Implementation overview:** This project assumes a basic understanding of geometric terminology (e.g. parallel, perpendicular, etc.), but it can serve as a good refresher to tighten up core understandings in these areas. Once students have been introduced to how to use a geoboard, they can be challenged to make the listed below. For those they believe not to be possible, students should work with peers to discuss possible solutions or to develop a mathematical justification for why such a construction is not possible.

- Two parallel lines
- Two perpendicular lines
- One line that doesn’t have a parallel
- One line that doesn’t have a perpendicular
- Two intersecting lines that don’t have a perpendicular
- Two lines that will intersect only if they are extended off the geoboard
- Two lines that intersect at more than one point

**Character components:** Intellectual character is developed as students test their understanding of core geometric concepts, and – in the extension activity below – as they use this understanding to develop original geometric constructions. Some of these original constructions they will know to be impossible, thus requiring application of a sophisticated understanding of the underlying principles. Performance character is
Developed as they are challenged to work at constructions that may appear to be impossible at first, testing multiple options before deciding if it is impossible. Civic character is developed as students maintain classroom norms of common inquiry and constructive response when they seek clarification or don’t agree. Moral character is developed as students build their own mathematical identity and by maintaining positive interpersonal relations with their peers and recognizing the talents and achievements of others.

**Extension:** Students can be given additional constructions to investigate, or challenged to develop their own sets, including both those that are possible and not possible.

**Sample Project: Grade 9**

**Algebraic Warmup**

**Student goal:** Many students new to algebra find the process of substituting letters for numbers to be baffling at first. This challenge helps students think algebraically with basic arithmetic concepts, and move forward to craft algebraic reasoning based on their success here.

**Implementation overview:** Students can work individually or in teams to crack the codes below. For each step, they should be challenged to explain their reasoning. For example, what (mathematically) do we know about B since it is the sum of two A’s? What are the possible options for D and F since it represents $A^2$?

\[
\begin{align*}
A + A &= B \\
A \times A &= DF \\
C + C &= DB \\
C \times C &= BD \\
A + C &= DE \\
A \times C &= EF
\end{align*}
\]

Once this puzzle has been solved, can students create their own and share it? This challenge should be framed to be original, and not just substituting different letters for the relationships embedded in this problem. Does their puzzle have only one answer or are there multiple answers? Can students use their math understanding to justify this claim?

**Character components:** Intellectual character is developed as students test their understanding of mathematical relationships and work toward comfort in representing numbers with letters. This is an important transition from arithmetic to algebraic thinking. Performance character is developed as students are challenged to work at problems iteratively to find a solution and to rule out alternatives. Civic character is developed as students maintain classroom norms of common inquiry and constructive
response when they seek clarification or don’t agree. Moral character is developed as students build their own mathematical identity and by maintaining positive interpersonal relations with their peers and recognizing the talents and achievements of others.

**Extension:** Can students construct an algebraic representation of a classic math problem such as chickens and cats in the barnyard? Given data that there are 32 heads and 86 legs, how many of each species can be found? Can students create their own problems? Challenge students to generate a problem with more than one correct solution. Is it possible? If such an arrangement is not possible, can they explain why not?

**Sample Project: Grade 12**

**Writing Simultaneous Equations**

**Student goal:** Students apply and extend their algebraic reasoning skills first by solving a simple problem involving simultaneous equations, and then through crafting their own. Developing original problems with unique solutions, and testing them on peers, develops classroom norms around shared mathematical inquiry.

**Implementation overview:** Students can work individually or in teams to develop algebraically valid representations that enable the following problem to be solved:

*For a group of friends buying a present, if each of them pays $8, they will have paid $3 too much. On the other hand, if each pays only $7, they will have paid $4 too little. How many friends are there?*

Once this puzzle has been solved, can students create their own and share it? This might be a simple re-working of the purchase scenario, or (better) an application of algebraic reasoning in a completely different field. Does their puzzle have only one answer or are there multiple answers? Can students use their math understanding to justify this claim?

**Character components:** Intellectual character is developed as students extend their basic understanding of algebraic representations and reasoning toward more complex problems. After a successful experience with the problem given, students are challenged to apply their reasoning skills in the development of original scenarios that are amenable to algebraic representations. Performance character is developed as students are challenged to work at problems iteratively to find a solution and to test its accuracy and completeness. Civic character is developed as students maintain classroom norms of common inquiry and constructive response when they seek clarification or don’t agree. Moral character is developed as students build their own mathematical identity and by
maintaining positive interpersonal relations with their peers and recognizing the talents and achievements of others.

**Extension:** Challenge students to solve a simultaneous equation problem using more than one method and express an opinion as to which, if either, is more useful in the context of that problem.

**Sample Project: Grade 12**

**Who Wins?**

**Student goal:** Students evaluate the fairness of a game in which two players move forward using different rules. If they find that the game is not fair, they are challenged to re-design it to enable an equal chance of winning.

**Implementation overview:** Students can work in pairs to play the following game: The first player moves forward three spaces each turn. The second player moves by rolling a standard 1-6 die and moving that many spaces. The first one to move 100 spaces wins. After playing the game a few times, students should have a handle on who will win most often. From here they can address two challenges:

- Would a shorter or longer game change the outcomes?
- How could the game be altered to ensure an equal chance of each player winning?

For both of these challenges, students should use their growing understanding of probability and statistics to justify their conclusions, not simply make assertions that are based only on the results of the games they played. Conclusions and recommendations can be made with appropriate support in the form of graphs and tables as students deem them to be useful.

**Character components:** Intellectual character is developed as students apply and test their understanding of basic probability concepts. Also, in making representations to justify their assertions about the fairness of the game (or the lack thereof), other basic math skills are employed intentionally, chosen for their applicability in this context. Performance character is developed as students are challenged to work “off script” in a problem that isn’t amenable to a simple worksheet exercise. Instead, they need to develop and apply an investigative scheme to assess the fairness of the game. Civic character is developed as students maintain classroom norms of common inquiry and constructive response when they seek clarification or don’t agree. Moral character is developed as students build their own mathematical identity and by maintaining positive interpersonal relations with their peers and recognizing the talents and achievements of others. This project also challenges students to consider issues of fairness in competition, and how rules can intentionally or otherwise bias a game in one
direction.

**Extension:** Can students alter the rules of another classic game to slightly bias it toward one player or another? Ideally, this would be a slight, almost undetectable bias.